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Optimisation of mono-articular or bi-articular linear actuation for a planar biped robot

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Robibio Project (RFI Atlanstic2020)

Design of a planar robot equipped with linear electric actuators

- Backdrivability property (no gear box) useful for impact



- How to place the linear actuators ?
 - Architecture
 - Attachment points

Actuation architecture

High level

- **mono-articular**
- **bi-articular**

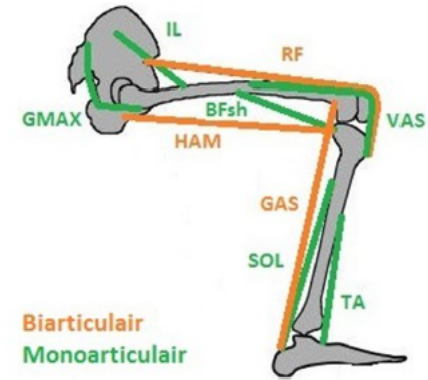


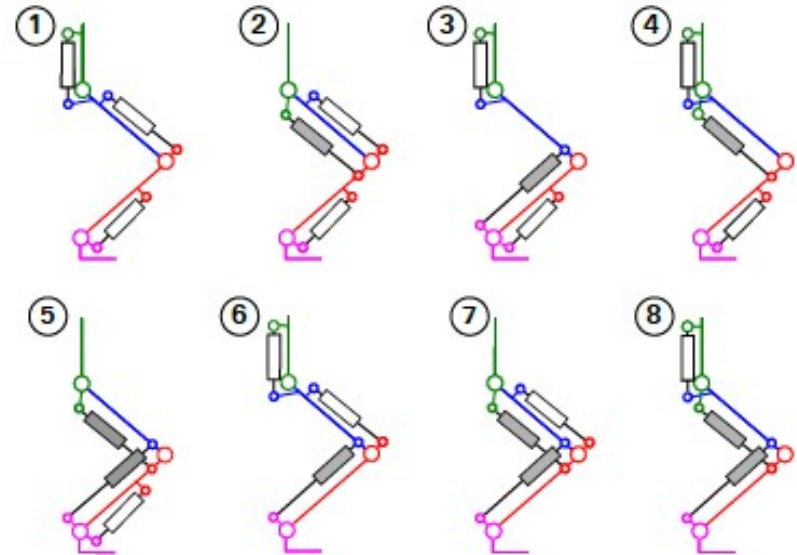
Figure 1. Main muscle groups of the lower limbs in the sagittal plane. Figure adapted from [30].

Robot with only 3 actuators in sagittal plane

3 of 5 actuators :

$$F_a, F_k, F_h, F_{ak}, F_{kh}$$

- 3 mono (case 1)
- 1 bi + 2 mono (2,3,4,6)
- 2 bi + 1 mono (5,7,8)



The model of a leg

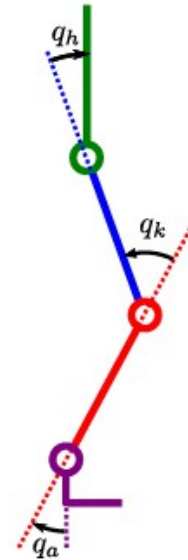
3 dof : q_a, q_k, q_h

The robot model

The dynamic model

$$A(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \Gamma$$

The torque is produced by the linear actuators (3 of 5)



$$\begin{bmatrix} \Gamma_a \\ \Gamma_k \\ \Gamma_h \end{bmatrix} = \begin{bmatrix} J_{a,a}(q_a) & J_{a,ak}(q_a, q_k) & 0 & 0 & 0 \\ 0 & J_{k,ak}(q_a, q_k) & J_{k,k}(q_k) & J_{k,kh}(q_k, q_h) & 0 \\ 0 & 0 & 0 & J_{h,kh}(q_k, q_h) & J_{h,h}(q_h) \end{bmatrix} \begin{bmatrix} F_a \\ F_{ak} \\ F_k \\ F_{kh} \\ F_h \end{bmatrix}.$$

Simplification of the model : the choice of actuators do not modify the mass distribution

Best attachment points of the motors

- Location of attachment points of a linear motor defines the torque available at the joint axis.
- With a linear motor, the torque available varies with joint axis.
- The robot is defined to achieve a set of given tasks
 - Squat motion
 - Walking as stance leg
 - Walking as swing leg

$$A(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \Gamma$$

➔ $D(t) = [t, q_a(t), q_k(t), q_h(t), \Gamma_a(t), \Gamma_k(t), \Gamma_h(t)]^T$.

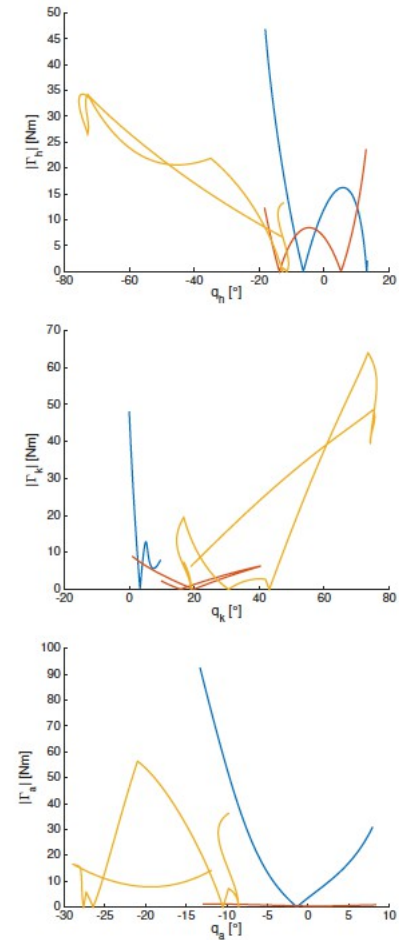


Fig. 3. Upper: hip torque against hip joint. Middle: knee torque against knee joint. Lower: ankle torque against ankle joint. Blue, red and yellow stand for stance/swing walking and squat, respectively.

Optimisation problem

- Constraints

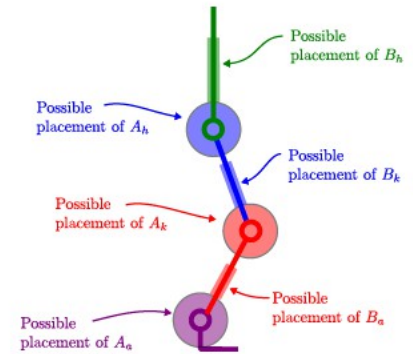
The attachment points belong to given areas

- Criterion

For design purposes, the objective is to find the best design that minimizes the maximal force

1 to 3 forces are considered simultaneously depending on architecture

Among the solutions minimizing C_1 , the minimal forces are expected to achieve the task :



$$C_1 = \min_{(A_1, B_1, A_2, B_2, A_3, B_3) \in S} (\max(|F_1|, |F_2|, |F_3|))$$

such that

$$\forall t, q_a(t), q_k(t), q_h(t), \Gamma_a(t), \Gamma_k(t), \Gamma_h(t) \in D(t),$$

$$\begin{bmatrix} \Gamma_a \\ \Gamma_k \\ \Gamma_h \end{bmatrix} = J(q_a, q_k, q_h) \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$C_2 = \min_{(A_1, B_1, A_2, B_2, A_3, B_3) \in S} \int_{t \in D} (F_1^2 + F_2^2 + F_3^2) dt$$

$$C_1 \rightarrow C = \mu C_1 + C_2 \text{ with large } \mu$$

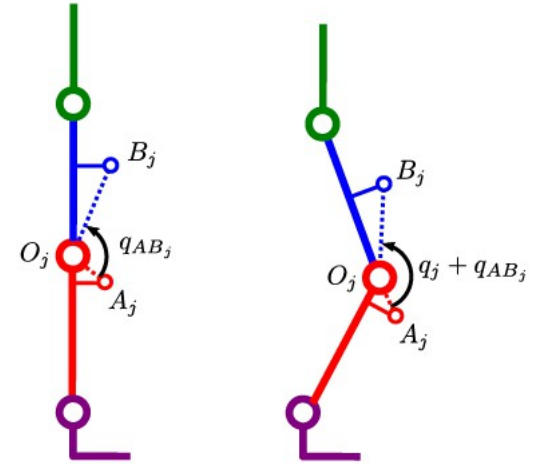
Design with mono-articular actuator

3 decoupled problems

Joint j $\Gamma_j = J_{j,j}(q_j)F_j$

$J_{j,j}$ is the height of the triangle $A_jO_jB_j$

$$J_{j,j}(q_j) = \frac{d_{A_j}d_{B_j} \sin(q_{AB_j} + q_j)}{\sqrt{d_{A_j}^2 + d_{B_j}^2 - 2d_{A_j}d_{B_j} \cos(q_{AB_j} + q_j)}}$$



- $J_{j,j}$ depends only on 3 parameters d_{A_j} , d_{B_j} , q_{AB_j} .
- $\sin(q_{AB_j} + q_j)$ at numerator \rightarrow singularity for a variation of $q_j > \pi$.
- q_{AB_j} is chosen to avoid singularity in workspace
- d_{A_j} , d_{B_j} have a similar contribution to $J_{j,j}$

$$\max_{0^\circ < q_{AB_j} + q_j < 180^\circ} J_{j,j}(q_j) = \min(d_{A_j}, d_{B_j})$$

Design for mono-articular actuator

Effects of parameters on $J_{j,j}$

- $\min(d_{Aj}, d_{Bj})$ is maximized in the possible placement area
- $\max(d_{Aj}, d_{Bj})$ changes the shape of $J_{j,j}$
- q_{ABj} translates the curve along horizontal axis

- $$F_{Mj} = \max_{\Gamma_j \in D} \frac{|\Gamma_j|}{\min(d_{Aj}, d_{Bj})}$$

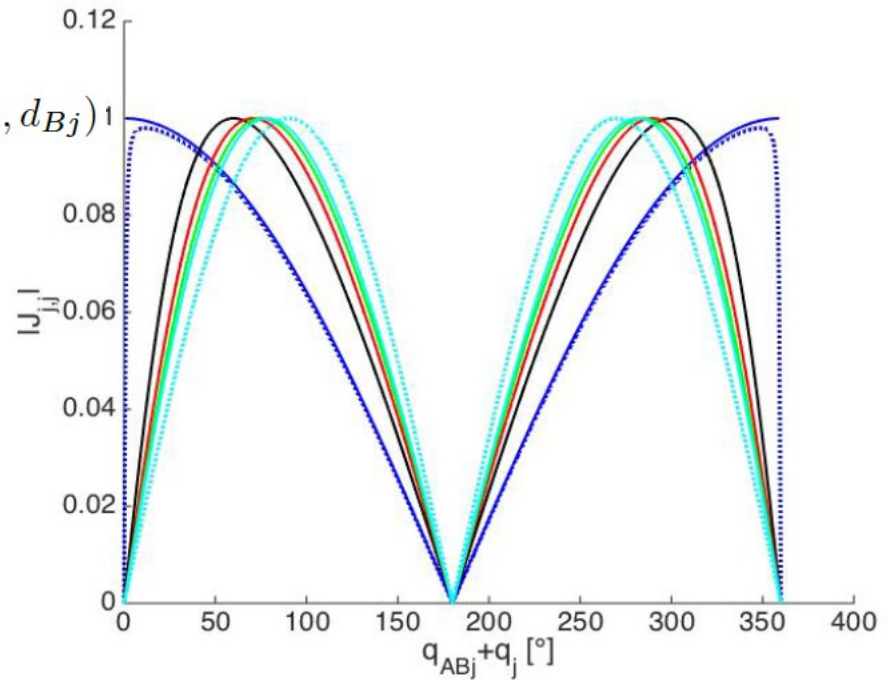
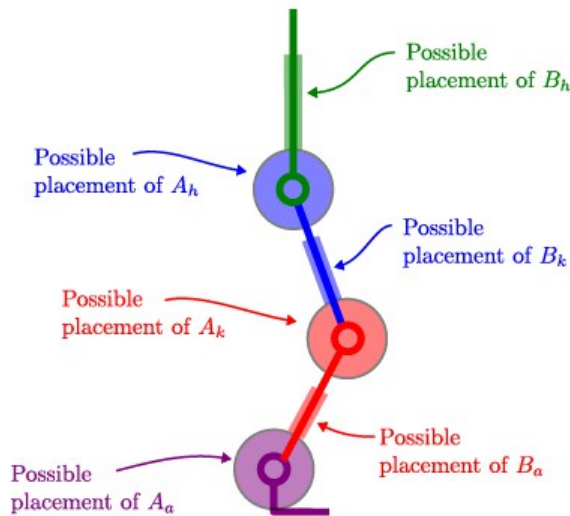


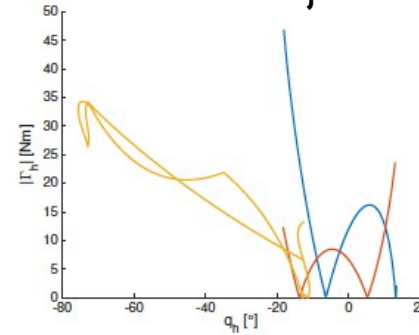
Fig. 5. Evolution of $|J_{j,j}|$ for $d_{Aj} = 0.1$ m for different values of d_{Bj} : $d_{Bj} = 0.098$ m (blue, dotted), $d_{Aj} = d_{Bj}$ (blue), $d_{Bj} = 0.2$ m (black), $d_{Bj} = 0.3$ m (red), $d_{Bj} = 0.4$ m (green), $d_{Bj} = 0.5$ m (cyan), $d_{Bj} = 1000$ m. There are two symmetric curves from 0° to 180° and from 180° to 360° . Their shape changes when d_{Bj} increases and becomes symmetrical in $q_{ABj} + q_j = 90^\circ$ and $q_{ABj} + q_j = 270^\circ$ when d_{Bj} tends to infinity.

Heuristic design for mono-articular actuator

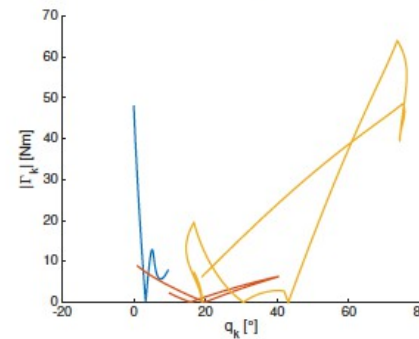
- The constraint $d_{A_j} < 0.1\text{m}$, $d_{B_j} < 0.4\text{m}$,



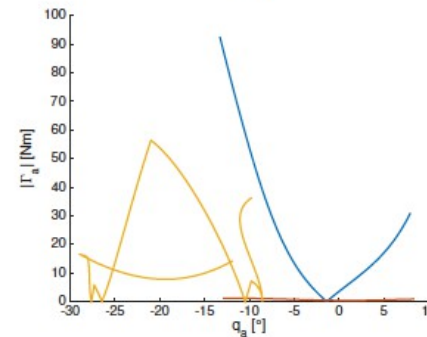
- The tasks
 - $d_{A_j} < 0.1\text{m}$, $d_{B_j} < 0.4\text{m}$,



The desired joint displacement $< 180^\circ$

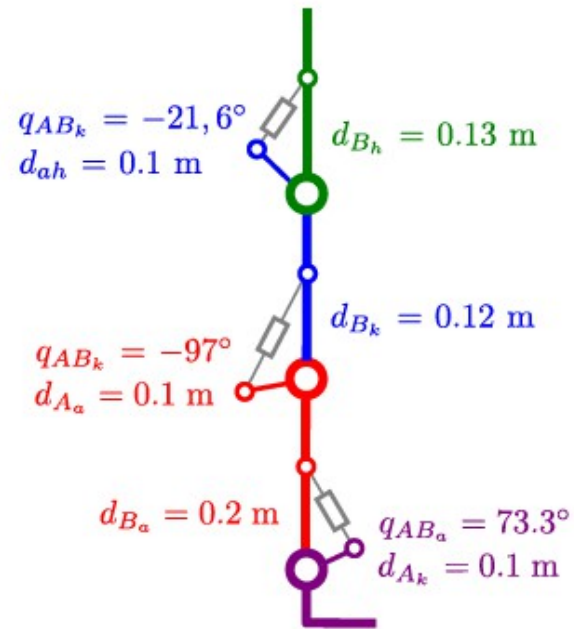
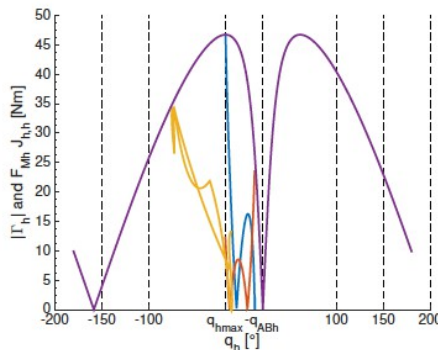
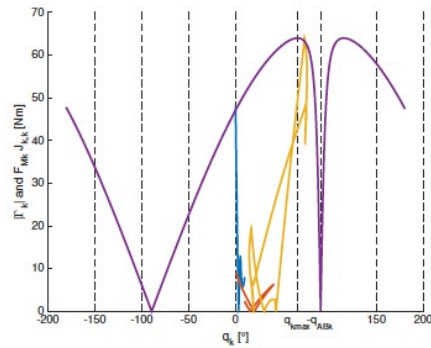
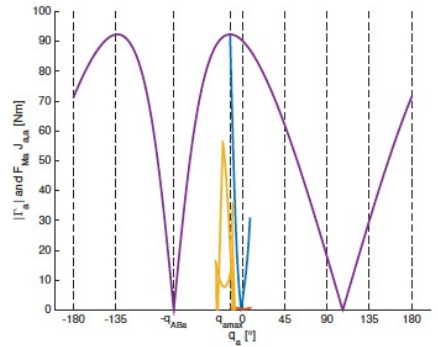


The higher required force is for ankle.



Heuristic design for mono-articular actuator

Optimisation of mono-articular or bi-articular linear actuation ...



Optimal design for mono-articular actuator

Optimisation can also be done numerically.

The parameters optimized are only d_{Bj} and q_{ABj} , $d_{Aj}=0.1\text{ m}$

The heuristic solution is the starting design for an optimisation done using SQP method.

The results are close for criterion : $C=\mu C_1 + C_2$

	Parameters	C_1 N	C_2 $10^5 N^2 s$
Method	Heuristic		
Ankle	$d_{Ba} = 0.2\text{ m}, q_{ABa} = 73.3^\circ$	922.9	1.63
Knee	$d_{Bk} = 0.11\text{ m}, q_{ABk} = -90.6^\circ$	651.2	1.66
Hip	$d_{Bh} = 0.13\text{ m}, q_{ABh} = -21.6^\circ$	467.3	1.32
Biped		922.9	4.62
Method	Numeric		
Ankle	$d_{Ba} = 0.17\text{ m}, q_{ABa} = 66.6^\circ$	922.9	1.63
Knee	$d_{Bk} = 0.11\text{ m}, q_{ABk} = -90.2^\circ$	649	1.66
Hip	$d_{Bh} = 0.14\text{ m}, q_{ABh} = -25.7^\circ$	467.3	1.31
Biped		922.9	4.61

Design of bi-articular actuator

- Four parameters : d_A , q_A , d_B , q_B
- The bi-articular actuator cannot be defined alone since in the architecture, several actuators act on the same joint

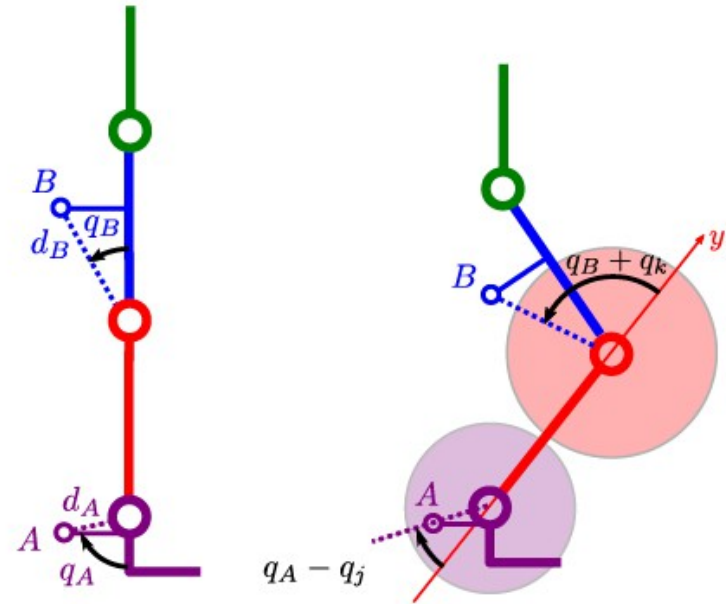
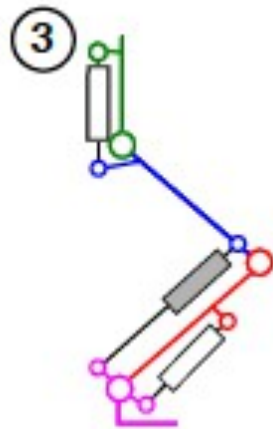


Fig. 10. Position of the attachment points for bi-articular actuation.

The bi-articular actuator acts alone on knee but not on ankle.

Bi-articular actuator F_{ak} is defined together with mono-articular actuator F_a

Mono-articular actuator F_h defined previously is optimal.

Heuristic design of bi-articular actuator

Model

$$\begin{bmatrix} \Gamma_a \\ \Gamma_k \end{bmatrix} = \begin{bmatrix} J_{aa}(q_a) & J_{a,ak}(q_a, q_k) \\ 0 & J_{k,ak}(q_a, q_k) \end{bmatrix} \begin{bmatrix} F_a \\ F_{ak} \end{bmatrix}$$

The bi-articular actuator acts on 2 joints with a ratio r_{ak}

$$\begin{bmatrix} \Gamma_a \\ \Gamma_k \end{bmatrix} = \begin{bmatrix} J_{aa}(q_a) & r_{ak}(q_a, q_k) J_{k,ak}(q_a, q_k) \\ 0 & J_{k,ak}(q_a, q_k) \end{bmatrix} \begin{bmatrix} F_a \\ F_{ak} \end{bmatrix}$$

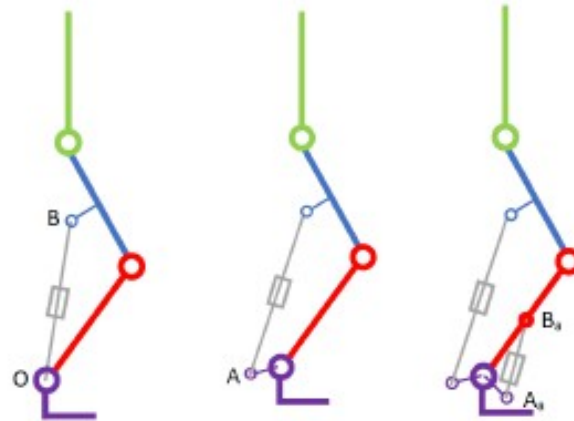
It can be written

$$\begin{bmatrix} \Gamma_a - r_{ak}\Gamma_k \\ \Gamma_k \end{bmatrix} = \begin{bmatrix} J_{aa}(q_a) & 0 \\ 0 & J_{k,ak}(q_a, q_k) \end{bmatrix} \begin{bmatrix} F_a \\ F_{ak} \end{bmatrix}$$

With a good design the load of the mono-articular ankle actuator can be reduced.

The bi-articular actuator actuates the knee alone.

Heuristic design of bi-articular actuator



The bi-articular motor M_{ak} is treated as a mono-articular one assuming that A is placed at the ankle joint center O .

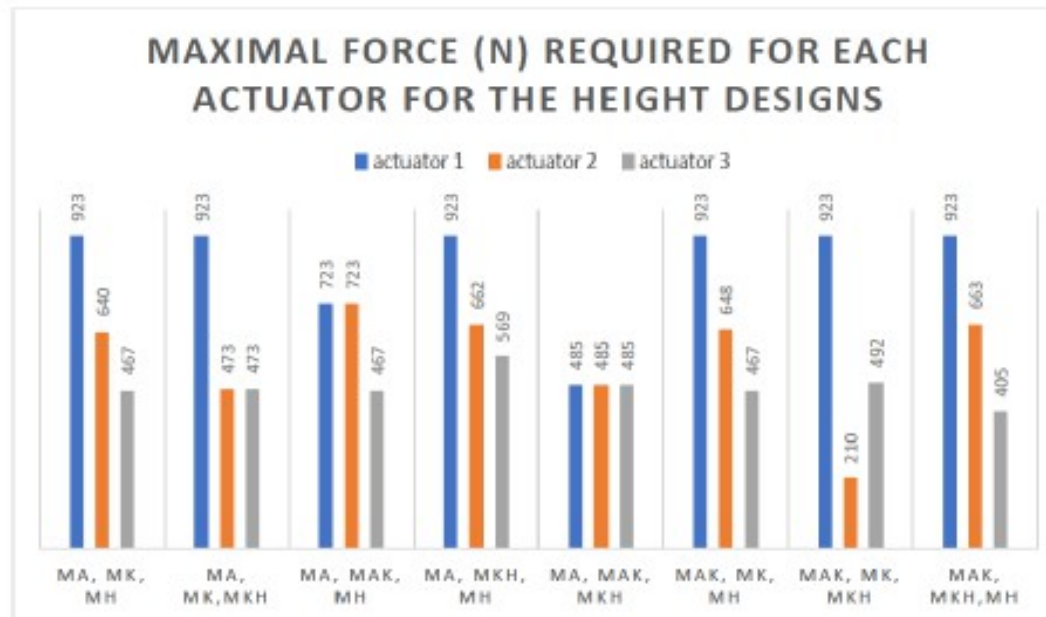
Choice of the pose of A in order to minimize $\Gamma_a - r_{ak}\Gamma_k$

The mono-articular ankle motor is designed to produce the torque $\Gamma_a - r_{ak}\Gamma_k$

The initial design can be improved by numerical optimisation.
 The methodology can be extended to the case of 2 bi-articular actuators

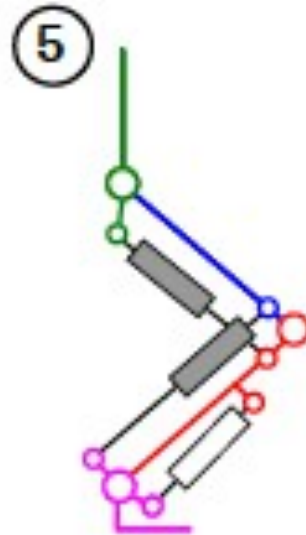
Comparison of the 8 designs

- The required torque is higher for the ankle.
- The maximal force at ankle is reduced when the mono-articular actuator is helped by a bi-articular actuator.
- Results are improved with 2 bi-articular actuators.

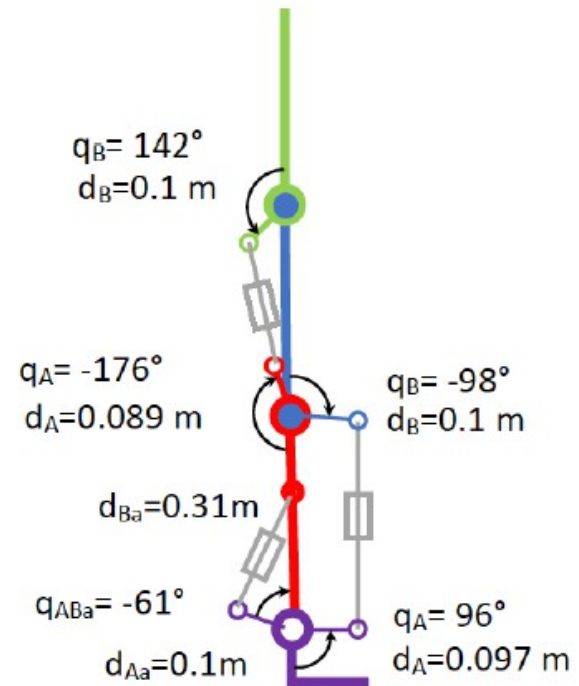


The optimal design

- The best architecture



- The best design



Conclusion/Perspective

- A methodology has been proposed to choose the best architecture of actuation with mono and bi-articular actuators.
- Bi-articular actuators allow to reduce the force used by actuators.
- A methodology has been proposed to choose the pose of attachment point for linear actuators.
- The design is based on the desired tasks to be achieved by the robot.
- A more complete set of tasks may lead to a different design
- It would be interesting to study redundant case with more than 3 actuators
- An extension to antagonist actuations can be considered (linear actuator can produce negative or positive forces contrary to muscle).