# The Effect of Vertical Motions on the Balance Recovery Prediction for Standing Humans through Linear MPC

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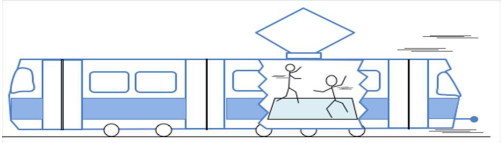

healthline.com

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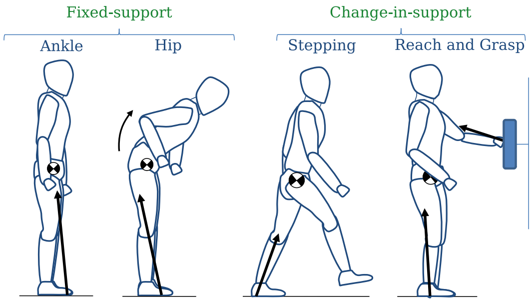
- emergency breaking in public transports
- person pushed down
- degradation of the physiological capacities due to ageing or disease.
- etc.

EquiSim Project: Digital platform for studying mechanisms (mechanics, biomechanics, sensory, cognitive, psychological) involved in the human balance preservation and recovery.

- Balance-recovery reactions




- The strategies to maintain balance
  - ankle strategy
  - hip strategy
  - stepping strategy

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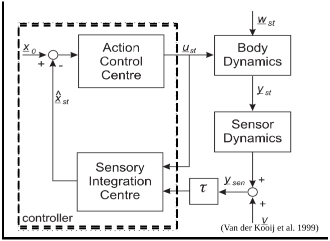
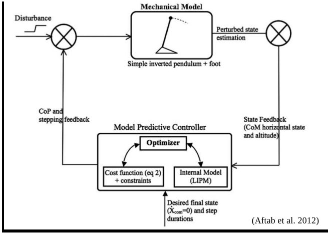



(Van der Kooij et al. 1999)




(Aftab et al. 2012)

- Regulation of the ankle strategy MAURER ET AL., 2006; MERGNER ET AL., 2003
- Regulation of the hip strategy ATKESON AND STEPHENS, 2007; PARK ET AL., 2004
- Using a predictive controller
  - choose actions over a time horizon AZEVEDO ET AL., 2002; DIEDAM ET AL., 2008
  - controls walking with optimized step placement HERDT ET AL, 2010
  - applied to balance recovery AFTAB ET AL, 2012
  - applied to balance recovery with proposition of a function based on the use of strategies VALLÉE ET AL, 2015

- Safe 3d bipedal walking through linear MPC with 3d capturability PAJON ET AL. 2019
    - Is their proposed mechanism relevant to use in balance recovery?
- Controller parameters for the young and elderly reactions AFTAB ET AL, 2012, VALLÉE ET AL, 2015

| Variable | Symbol | Value | Ref. |
| --- | --- | --- | --- |
| Body height | $H$ | 1.75 m | Chaffin and Andersson (1984) |
| Body mass | $m$ | 75 kg | Chaffin and Andersson (1984) |
| CoM height | $h = 0.575 \times H$ | 1.012 m | Winter (1990) |
| Foot length | $l_f = 0.152 \times H$ | 0.26 m | Winter (1990) |
| Ankle to toe distance | $0.81 \times l_f$ | 0.21 m | Winter (1990) |
| Ankle to heel distance | $0.19 \times l_f$ | 0.05 m | Winter (1990) |
| Max trunk rotation | $\theta_{max}$ (forward) | $\pi/2$ rad | |
| Min trunk rotation | $\theta_{min}$ (backward) | $-\pi/2$ rad | |
| Trunk inertia | $j$ | 8 kg.m$^2$ | Winter (1990) |
| Max hip torque | $\tau_{max}$ | 190 N.m | Chaffin and Andersson (1984) |

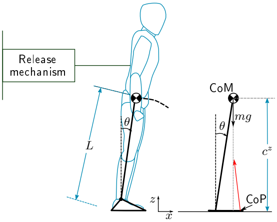
| Parameter | Value |
| --- | --- |
| Body Height, H | 1.57 m |
| Body Mass | 65 kg |
| Foot length, $l_f$ | 0.152 × H |
| Step preparation time, $T_{prep}$ | 150 ms (fixed) |
| Reaction time, $T_{reac}$ | Varied between 75 and 150 ms |
| Leg swing time, $T_{step}$ | Chosen by the controller |
| Max foot acceleration, $\ddot{f}'_{max}$ | Varied between 60 and 180 m.s$^{-2}$ |
| Max trunk rotation, $\theta_{max}$ | $\pi/3$ rad |
| Max hip torque, $\tau_{max}$ | 100 N.m |

- based on Newton-Euler equations of motion of the whole human

$$\boldsymbol{p}^{x,y} = \boldsymbol{c}^{x,y} - \zeta \ddot{\boldsymbol{c}}^{x,y} \in \text{conv}\{s_i\}$$

$$\zeta = \frac{\boldsymbol{c}^z - \boldsymbol{p}^z}{\ddot{\boldsymbol{c}}^z + g}$$

$c$ Center of the Mass (CoM) position, $p$ Center of the Pressure (CoP)
$\ddot{c}$ acceleration of CoM, $\zeta$ nonlinear term, $^{x,y,z}$ are coordinates of 3D vector
where $^z$ is a vertical coordinate
and conv$\{s_i\}$ is the convex hull of the support foot $i$



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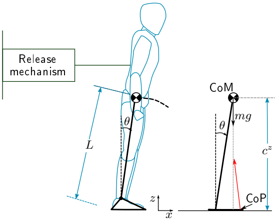


- The variation of $\zeta$ can be reasonably bounded during stepping motions

$$\underline{\zeta} \leqslant \zeta \leqslant \overline{\zeta} \quad \text{Brasseur 2015}$$

# Materials and methods



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$$\boldsymbol{p}^{x,y} = \boldsymbol{c}^{x,y} - \zeta \ddot{\boldsymbol{c}}^{x,y} \in \mathrm{conv}\{s_i\}$$

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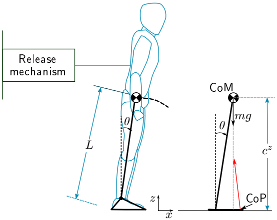


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- so we only have to check that

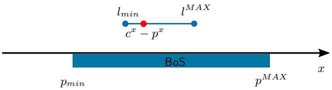
$$\{\ddot{\boldsymbol{c}}^{x,y} - \underline{\zeta}\ddot{\boldsymbol{c}}^{x,y}, \ddot{\boldsymbol{c}}^{x,y} - \overline{\zeta}\ddot{\boldsymbol{c}}^{x,y}\} \in \mathrm{conv}\{s_i\}$$

- we impose that the vertical motion of the CoM satisfies

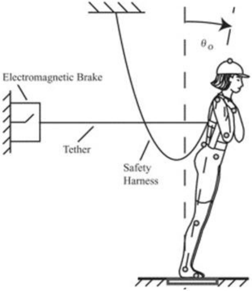
$$\underline{\zeta}(\ddot{\boldsymbol{c}}^z + g) \leqslant \boldsymbol{c}^z - \boldsymbol{p}^z \leqslant \overline{\zeta}(\ddot{\boldsymbol{c}}^z + g)$$

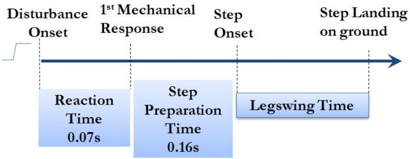
- which is equivalent to

$$l_{min}(\underline{\zeta}\ddot{c}^x) \leqslant c^x - p^x \leqslant l^{MAX}(\overline{\zeta}\ddot{c}^x)$$




- Single step recovery
  - inputs: lean angle $\theta$, average subject anthropometry, step timings
  - comparison: maximum release angle vs vertical motion



- MPC objective function formulation

$$\text{minimize} \quad \sum_{k=m}^{M} \mu_1 d_1(t_k) + \mu_2 d_2(t_k) + \mu_3 d_3(t_k) + \mu_4 d_4(t_k) \qquad (1)$$

$$\text{subject to} \quad (3) - (9) \text{ at all } t_k \in [t_m, t_M], \qquad (2)$$

$\mu_1, \mu_2, \ldots, \mu_4$ are the weights

minimizing the jerk parameter $\delta_k$ for the piecewise polynomial solutions

$$d_1 = \|\delta_k\|^2$$

The piecewise polynomial solutions are of the form: $\boldsymbol{c}(t) = \alpha_k' + \beta_k' \tau_k + \frac{\gamma_k'}{2}\tau_k^2 + \frac{\delta_k'}{6}\tau_k^3$ with $\tau_k = t - t_k$ and constant parameters $\alpha_k'$, $\beta_k'$, $\gamma_k'$ and $\delta_k'$.

minimizing the deviation:

$$d_2 = \left\|\dot{\boldsymbol{c}}^x - \dot{\boldsymbol{c}}^x_{ref}\right\|^2; \quad d_3 = \left\|\dot{\boldsymbol{c}}^y - \dot{\boldsymbol{c}}^y_{ref}\right\|^2$$

$\dot{\boldsymbol{c}}^{x,y}_{ref}$ is the horizontal reference speed of CoM

keeping the CoP as close as possible to the center of the support foot $i$

$$d_4 = \left\|\boldsymbol{c}^{x,y} - \frac{1}{2}(\overline{\zeta} + \underline{\zeta})\ddot{\boldsymbol{c}}^{x,y} - (s_i^{x,y} + \sigma_i^{x,y})\right\|^2$$

$\sigma_i^{x,y}$ is the horizontal translation from the ankle position $s_i^{x,y}$ to the foot center

- Dynamic feasibility:

The CoP must lie in the convex hull of contact points

$$\{\boldsymbol{c}^{x,y} - \overline{\zeta}\ddot{\boldsymbol{c}}^{x,y}, \boldsymbol{c}^{x,y} - \underline{\zeta}\ddot{\boldsymbol{c}}^{x,y}\} \in s_i^{x,y} + \mathcal{S} \tag{3}$$

$s_i^{x,y} + \mathcal{S}$ is the convex hull of the support foot $i$

The vertical motion of the CoM satisfies

$$\underline{\zeta}(\ddot{\boldsymbol{c}}^z + g) \leq \boldsymbol{c}^z - \boldsymbol{p}^z \leq \overline{\zeta}(\ddot{\boldsymbol{c}}^z + g) \tag{4}$$

with the hypothesis of no free falling

$$\ddot{\boldsymbol{c}}^z + g \geq 0 \tag{5}$$

and maximum swing leg acceleration

$$|\ddot{\boldsymbol{f}}| \leq \ddot{f}_{max} \tag{6}$$

- Kinematic feasibility:

The maximum reachable region of the hip $A_i(\boldsymbol{c} - s_i) \leq b_i$ (7)

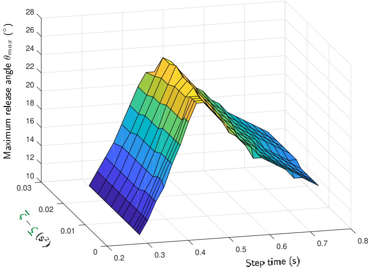
with some fixed matrix $A_i$ and vector $b_i$ linked to support foot $i$

maximum reachable region of foot $i$ $\begin{array}{rcl} A_{i+1}(\boldsymbol{c}(t_i) - s_{i+1}) & \leq & b_{i+1} \\ A_i(\boldsymbol{c}(t_{i+1}) - s_i) & \leq & b_i \end{array}$ (8)

avoiding the foot overlay $\underline{S}^{x,y} \leq s_{i+1}^{x,y} - s_i^{x,y} \leq \overline{S}^{x,y}$ (9)

## one-step balance recovery strategy

- maximum release anlge is about 23°
- optimal step duration of about 440ms
- variation of the size $(\bar{\zeta} - \underline{\zeta})$ has almost no effect




- largely perturbed condition:
  - increasing the $(\underline{\zeta}, \bar{\zeta})$ interval leads to a more constrained displacement of the CoP
  - decreasing it leads to a more constrained vertical dynamics of the WBCoM

# Conclusions

- Relevance
  - linear model predictive control scheme for bipedal walking and balance recovery
  - non-constraint vertical dynamics of the WBCoM
- Findings
- Limitations

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  - relatively small vertical motion of the WBCoM using high dynamics
  - proposed approach provide similar results to those obtained with a classical LIPM
  - results obtained match quiet closely those observed in humans for both slightly and largely perturbed situations (walking and extreme balance recovery)
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  - results obtained match quiet closely those observed in humans for both slightly and largely perturbed situations (walking and extreme balance recovery)
- Limitations
  - evaluating the method using different external physical model and multi-contact problems not considered
  - non-linear approach not covered