An original walking composed of a ballistic single-support and a finite time double-support motions.

JNRH 2020,
26-27 June 2020

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June 24, 2020

Motivation and strategy

Motivation.

- To define a walking motion with single support motion and a finite time double support motion.
- The planar biped has two identical legs, a trunk and two massless feet.

Strategy.

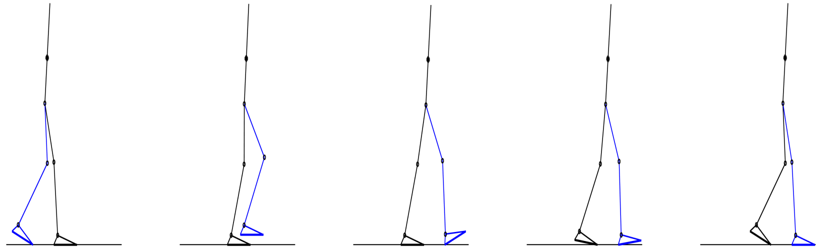
- The motion of the single support is defined with a boundary value problem combining position and speed conditions.
- During the single ankle torque controls the motion of the *CoP*
- During the double support the two feet rotate.

Presentation outline

Statement of problem.

- The walking motion.
- Mathematical model and design of the walking.
- Single-support motion: Boundary value problem with condition in position and velocities.
- Double-support motion: problem definition.
- Numerical results.

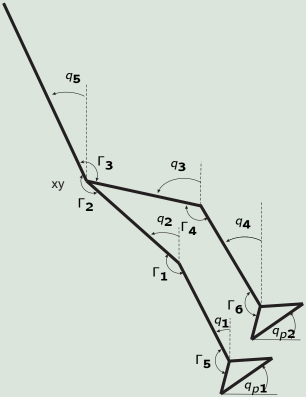
A step with a single support motion and a double support motion.

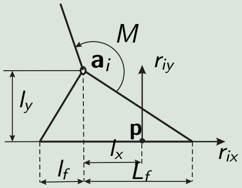


a) b) c) d) e)

a) Initial configuration of biped in single support. b) Intermediate configuration in single support. c) Final configuration in single support – initial one in double support. d) Intermediate configuration in double support. e) Final configuration in double support – initial one in the next single-support motion.

Scheme of the biped and the assumption of the massless foot.





- assumption of the massless foot The weight of a humain foot is almost 0.600 kg $\Rightarrow$ Their inertial effects are neglected.

Mathematical model 2/2.

The dynamic motion and unilateral constraints.

- generalized vector
  $\mathbf{x} = [q_1, q_2, q_3, q_4, q_5, x, y]^\top$.
- Mathematical model of the biped

$$\mathbf{A}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{H}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + g\mathbf{G}(\mathbf{x}) = \mathbf{D}\Gamma + \mathbf{J}_{\mathbf{r_1}}^\top \mathbf{r}_1 + \mathbf{J}_{\mathbf{r_2}}^\top \mathbf{r}_2, \qquad (1)$$

- The following constraint equations are correct when the front or/and rear leg is/are on the bearing surface.

$$\mathbf{J}_{\mathbf{r}_i}\ddot{\mathbf{x}} + \dot{\mathbf{J}}_{\mathbf{r}_i}\dot{\mathbf{x}} = \mathbf{0} \qquad \text{for } i = 1 \text{ or/and } 2. \qquad (2)$$

The single support 1/3

The main characteristics of the single support.

- Let leg 1 be the supporting leg:

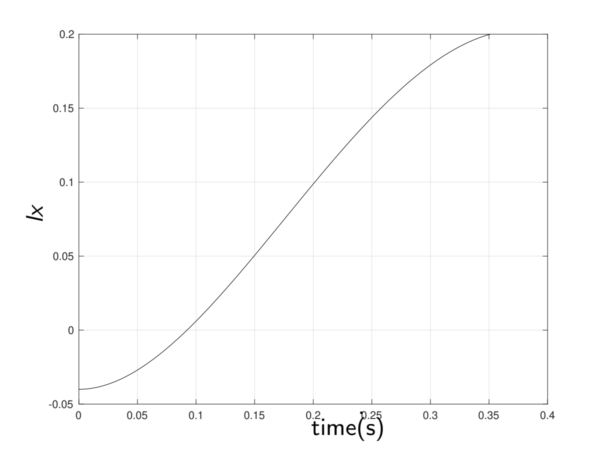
$$\begin{array}{lcll} \mathbf{r}_2 & = & \mathbf{0}. & \text{No action ground reaction in foot 2} \\ \Gamma_i & = & 0 \ (i = 1, 2, 3, 4, \ \text{and } 6). & \text{Ballistic motion} \\ \dot{V}_{\mathbf{a}_1} & = & \mathbf{0}. & \text{The stance foot should not move} \\ \Gamma_5 & = & l_x r_{1y} + l_y r_{1x} & \text{To control the trajectory CoP}. \end{array} \quad (3)$$

- Nine scalar equations (model and constraints) to find the unknown variables ($\ddot{\mathbf{x}}$, Γ_5, and the two components of $\mathbf{r}_1$). $\Rightarrow$ To overcome this difficulty The trajectory of the CoP is defined with a polynomial function in time $l_x(t)$.

$$\begin{bmatrix} \mathbf{A}(\mathbf{x}) & -\mathbf{d}_5 l_y + \mathbf{j}_{\mathbf{r}_{11}}^\top & -\mathbf{d}_5 l_x + \mathbf{j}_{\mathbf{r}_{12}}^\top \\ \mathbf{J}_{\mathbf{r}_1} & \mathbf{0}_{2\times 1} & \mathbf{0}_{2\times 1} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}} \\ r_{1x} \\ r_{1y} \end{bmatrix} = \begin{bmatrix} -\mathbf{H}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} - g\mathbf{G}(\mathbf{x}) \\ -\dot{\mathbf{J}}_{\mathbf{r}_1}\dot{\mathbf{x}} \end{bmatrix}, \quad (4)$$

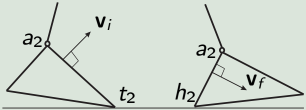
The single support 2/3

The trajectory of the CoP from the heel to the toe.



The Boundary value problem.

- The five initial velocities $\dot{q}_i(0)$, length of the step d and duration T_{SS} of the single-support motion are calculated to reach the final configuration from the initial one and satisfying two velocities conditions.



- To avoid improper contact with the terrain at the initial moment of single support.

$$V_{\mathbf{a}_2}(0) \cdot \mathbf{a_2 t_2}(0) = 0 \quad (5)$$

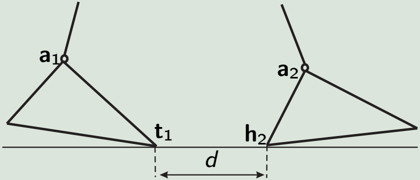
- No impact at the end of the single support.

$$V_{\mathbf{a}_2}(T_{SS}) \cdot \mathbf{a_2 h_2}(T_{SS}) = 0 \quad (6)$$

Double support motion: 1/2

The main characteristics of the double support motion.

- The two feet rotate:



- Trajectories of $\mathbf{a_1}$ and $\mathbf{a_2}$ are circles: Knowledge of q_{p1} and q_{p2} $\Rightarrow$ $x_{\mathbf{a_1}}$, $y_{\mathbf{a_1}}$, $x_{\mathbf{a_1}}$, and $y_{\mathbf{a_2}}$.
- Knowledge of $x_{\mathbf{a_1}}$, $y_{\mathbf{a_1}}$, q_1, and q_2 $\Rightarrow$ x and y (DGM).
- Knowledge of x, y, $x_{\mathbf{a_2}}$, and $y_{\mathbf{a_2}}$ $\Rightarrow$ q_3 and q_4 (IGM).
- Variables q_{p1}, q_{p2} (orientations of feet), q_1, q_2, q_5, are defined with polynomial functions in time.
- 7 scalar equations; 10 unknown equations (6 Torques and 4 components of $\mathbf{r}_1$ and $\mathbf{r}_2$).

Equation to describe the double support motion.

- Equilibrium equations of the feet

$$\Gamma_5 = (x_{\mathbf{t_1}} - x_{\mathbf{a_1}})r_{1y} - (y_{\mathbf{t_1}} - y_{\mathbf{a_1}})r_{1x}, \quad \Gamma_6 = (x_{\mathbf{h_2}} - x_{\mathbf{a_2}})r_{2y} - (y_{\mathbf{h_2}} - y_{\mathbf{a_2}})r_{2x}.$$

- The horizontal component r_{1x} is defined with polynomial functions in time.
- $\Rightarrow$ Motion equation.

$$\mathbf{A}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{H}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + g\mathbf{G}(\mathbf{x}) - (\mathbf{j}_{\mathbf{r_{11}}}^{\top} - \mathbf{d}_5(y_{\mathbf{t_1}} - y_{\mathbf{a_1}}))r_{1x} =$$

$$\begin{bmatrix} \mathbf{D}_{14} & \mathbf{j}_{\mathbf{r_{12}}}^{\top} + \mathbf{d}_5(x_{\mathbf{t_1}} - x_{\mathbf{a_1}}) & \mathbf{J}_{\mathbf{r_2}}^{\top} + \mathbf{d}_6 \begin{bmatrix} -(y_{\mathbf{h_2}} - y_{\mathbf{a_2}}) & x_{\mathbf{h_2}} - x_{\mathbf{a_2}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \\ r_{1y} \\ r_{2x} \\ r_{2y} \end{bmatrix},$$

Numerical parameters 2/2.

Physical parameters close to those of humans.

	Mass (kg)	Length (m)	Inertia moment ($kg \cdot m^2$)	Center of mass (m)
Human shin	$m_s = 4.6$	$l_s = 0.55$	$I_s = 0.0521$	$s_s = 0.324$
Human thigh	$m_t = 8.6$	$l_t = 0.45$	$I_t = 0.75$	$s_t = 0.18$
Human trunk	$m_T = 48.6$	$l_p = 0.75$	$I_T = 11.3$	$s_T = 0.386$
Foot, *COP* excursion	$m_f = 0.0$	$l_f = 0.04$ $L_f = 0.07$ $l_y = 0.07$	$I_f = 0.00$	

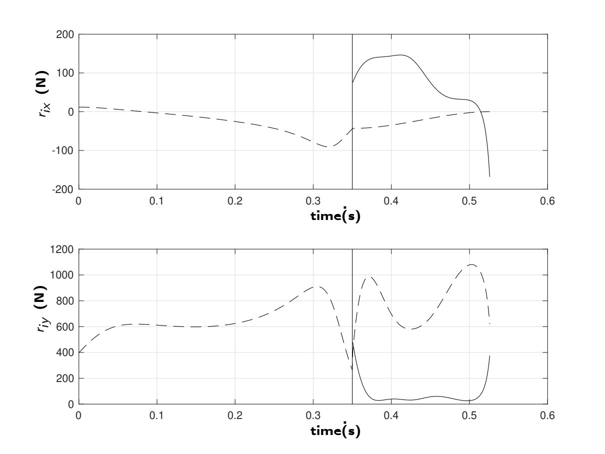
Numerical parameters 2/2.

The durations of the two phases.

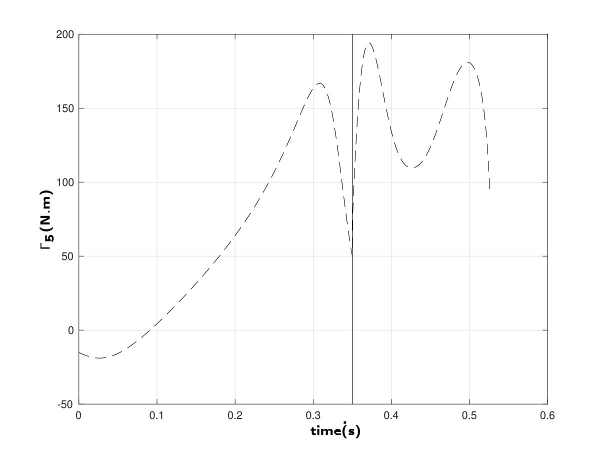
- $T_{SS} = 0.3506$ s; $T_{DS} = 0.1753$ s ($T_{DS} = \frac{T_{SS}}{2}$).
- For T_DS values lower than $\frac{T_{SS}}{2}$: Impossible to obtain a double support motion that satisfies the unilateral constraints.
- Percentage of the finite time double support motion, with respect to the complete cycle, which corresponds to two walking step: 17%
- This percentage is coherent with human data, see
  - Rose, J., and Gamble, G. Human Walking, Third Edition. Lippincott Williams & Wilkins, 2006.
  - Winter, W. Biomechanics and Motor Control of Human Movement. Fourth Edition, John Wiley & Sons, Inc., 2009.

Numerical results 1/6

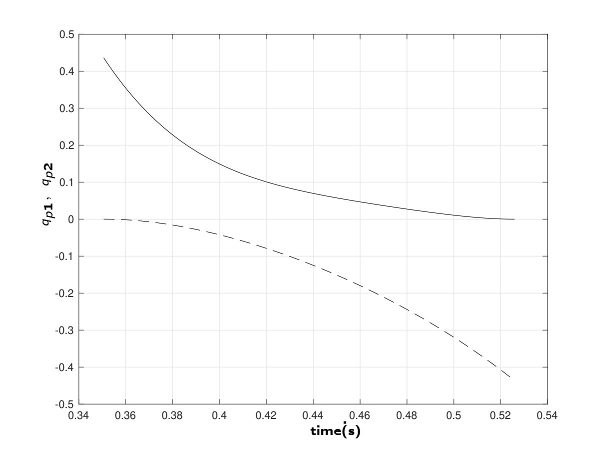
The ground reactions acting in the two feet during the walking step.



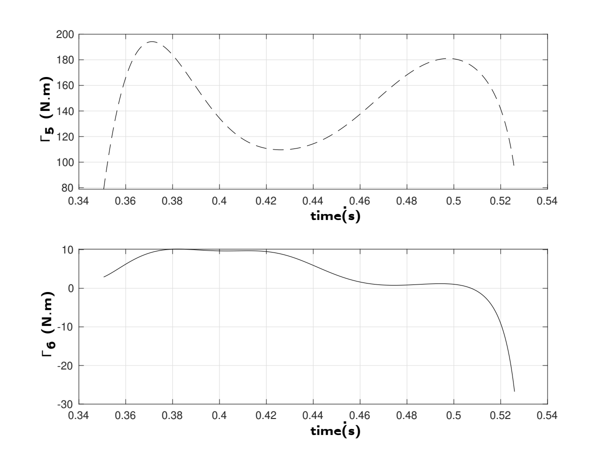
Torque in the leg 1 during the walking step.



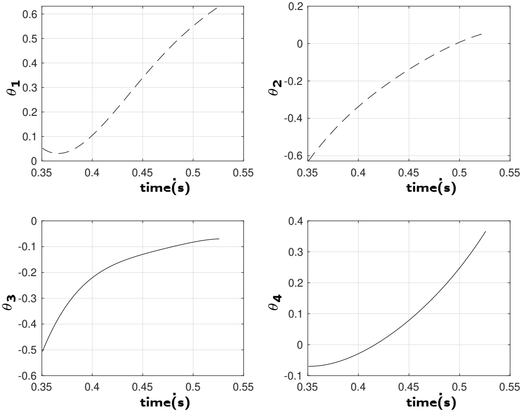
Orientation of the two feet during the double support motion.



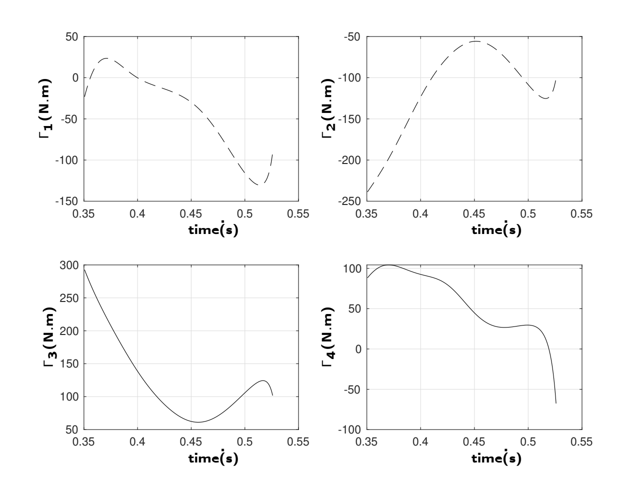
Torques in the two ankles during the double support motion.



Joint angles in knees and hips for the two legs.



Torques in knees and hips of two legs during the double support motion.



Conclusion and perspectives 1/2

Conclusion

- A ballistic motion is define to design the single support..
- A torque applied in the ankle joint of the stance foot controls a trajectory of the CoP under the sole.
- Thanks to the conditions of the velocities we can any collision of the transfer leg with the ground and an impact at the end of the single support.
- A *distribution in time* of the double support allows to get a cyclic gait with the torques of finite magnitude.

Perspectives

- One objective is to explore a starting phase that beginning with sequence of a finite time double support and a ballistic walking motion to reach a periodic motion and its dual problem with a stopping phase.
- A second objective is to extend the study about a passive exoskeleton to investigate how the reduce the torques proved by human.

Thank you for your attention.