

# An original walking composed of a ballistic single-support and a finite time double-support phases

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**Summary.** The paper aim is to define an original walking for a 2D biped with a trunk, two identical legs with knees and massless feet. This walking is composed of a ballistic single-support phase and a distributed in time double-support phase. The ballistic movement in single support is defined by solving a boundary value problem with initial and final biped configurations and velocity conditions. These conditions ensure that at the beginning of the single support the toe of the rear leg rises without touching the ground again and at the landing of the heel there is no impact. In the double-support phase, the orientation of the two feet and other generalized coordinates which are used to define the configuration of the biped, are chosen as Bezier functions of time. The torques and ground reaction forces resulting from this double-support phase are determined by solving for the biped the inverse dynamic problem.

## Statement of the problem

### The walking motion

We design biped periodic walking, which consist of distributed in time single- and double-support phases. Ballistic single-support motion is designed. During this motion the torques in all joints are zeroes except the torque in the ankle-joint of the stance leg. The torque in the ankle-joint of the stance leg is applied in order to keep its foot in the equilibrium. During double-support motion the torques are applied in all the six joints; during this time both feet rotate: foot of the rear leg - around its toe, foot of the front leg - around its heel. To explain our statement of the problem more clearly, we show Fig. 1 with several stick-figures, which results from our numerical investigations.

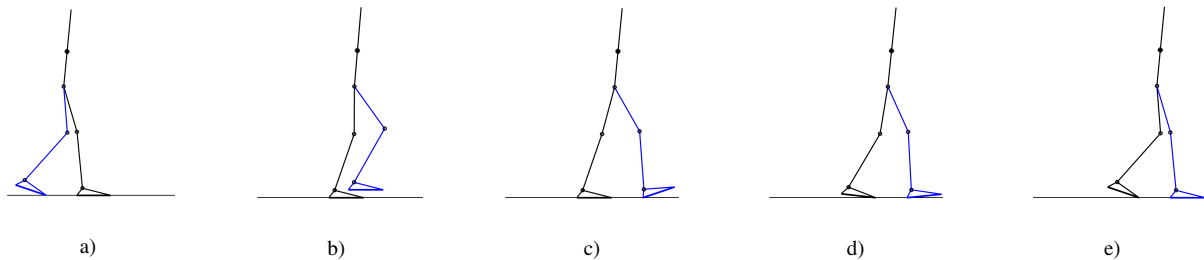


Figure 1: a) Initial configuration of biped in single support. b) Intermediate configuration in single support. c) Final configuration in single support – initial one in double support. d) Intermediate configuration in double support. e) Final configuration in double support – initial one in the next single-support motion.

### Mathematical model and design of the walking

The absolute orientation of the shins and thighs are defined with angles  $q_1, q_2, q_3, q_4$  (see Fig. 2). The orientation of the trunk is defined by angle  $q_5$ . Cartesian hip-joint coordinates are  $x$  and  $y$ . The orientation of both feet are described by angles  $q_{p1}$  and  $q_{p2}$ . It is assumed that there are no sliding motion and no take-off of the support legs. In real human walking, the double-support motion is distributed in time, and consequently the configuration of the human at the beginning of the double support differs from the configuration at the end of this double support. Consequently for the human walking the configurations at the beginning and at the end of the single-support motion are also different. Vector  $\mathbf{x}$  of the generalized coordinates for the biped with massless feet is  $\mathbf{x} = [q_1, q_2, q_3, q_4, q_5, x, y]^\top$ . Superscript  $\top$  means transposition. The mathematical model of the biped is:

$$\mathbf{A}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{h}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{D}\boldsymbol{\Gamma} + \mathbf{J}_{\mathbf{r}_1}^\top \mathbf{r}_1 + \mathbf{J}_{\mathbf{r}_2}^\top \mathbf{r}_2, \quad (1)$$

where  $\mathbf{A}(\mathbf{x})$  is a  $7 \times 7$  symmetric positive definite inertia matrix,  $\mathbf{h}(\mathbf{x}, \dot{\mathbf{x}})$  is a  $7 \times 1$  vector, which groups the centrifugal, Coriolis, and gravity forces.  $\boldsymbol{\Gamma}$  is  $6 \times 1$  vector of the joint torques applied by the biped. We consider six torques applied in the hip-, knee- and ankle-joints. Vectors  $\mathbf{r}_i = (r_{ix}, r_{iy})^\top$ , with  $i = 1, 2$ , are the ground reactions applied to the massless feet and consequently to the ankle-joints. The following constraint equations are correct when the front or/and rear leg is/are on the bearing surface.

$$\mathbf{J}_{\mathbf{r}_i} \ddot{\mathbf{x}} + \dot{\mathbf{J}}_{\mathbf{r}_i} \dot{\mathbf{x}} = \mathbf{0} \quad \text{for } i = 1 \text{ or/and } 2. \quad (2)$$

### Single-support motion: Boundary value problem

In the single-support phase, the ballistic movement takes place on the supporting leg with a flat-foot contact, Fig. 1. A torque is applied only in the ankle-joint of this stance leg in order to maintain the equilibrium of its foot. Let  $a_2, t_2$ , and  $h_2$  refer to the ankle, toe and heel of the transferred leg (shown in Fig. 1 by blue color). The five initial velocities  $\dot{q}_i(0)$ ,

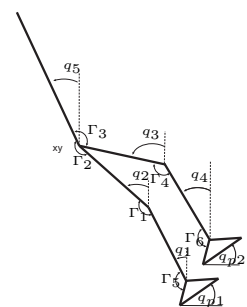


Figure 2: Generalized coordinates, and inter-link torques.

length of the step  $d$  and duration  $T_{SS}$  of the single-support motion are calculated to reach the final configuration from the initial one and to satisfy the two following velocity conditions  $V_{a_2}(0) \cdot \mathbf{a}_2 \mathbf{t}_2 = 0$  and  $V_{a_2}(T_{SS}) \cdot \mathbf{a}_2 \mathbf{h}_2 = 0$ . They respectively ensure that the toe  $t_2$  of the rear foot rises from the ground without touching the ground again and there is an impactless landing of the heel  $h_2$  on the ground at the end of the single-support motion.

**Double-support motion: problem definition**

At the end of the single-support phase, the landing massless foot touches the ground with its heel, Fig. 1 c). The other foot keeps contact with the ground through its toe. In double-support phase both feet rotate, Fig. 1 d), the double support motion is ended when the foot of the front leg touches the ground with the toe and the rear foot rises. At this instant, the single-support motion of the next step starts. Variables  $q_{p1}$ ,  $q_{p2}$ ,  $q_1$ ,  $q_2$ ,  $q_5$  and  $r_{1x}$  are defined with Bezier polynomial functions  $P_j(\tau)$  with five control point  $p_j$  ( $j = 0, \dots, 4$ ) and  $\tau = (t - T_{SS})/T_{DS}$ ,  $T_{DS}$  being the duration of the double-support motion. At each sampling time the knowledge of  $P_j(\tau)$  allows us to calculate the trajectory of  $x$ ,  $y$ ,  $x_{a1}$ ,  $x_{a2}$ ,  $q_3$  and  $q_4$ , by using geometric and kinematic models, (2), and thus the left term of (1).

**Numerical results**

Figure 3 a) shows that the evolutions of angles  $q_{p1}$  and  $q_{p2}$  are synchronized during the double-support phase. Figure 3 b) shows that the magnitude of the torque in the ankle-joint of the rear leg is much greater than the magnitude of the torque in the ankle-joint of the front leg. This seems physically coherent because the biped pushes on the rear leg to move forward during the double-support phase.

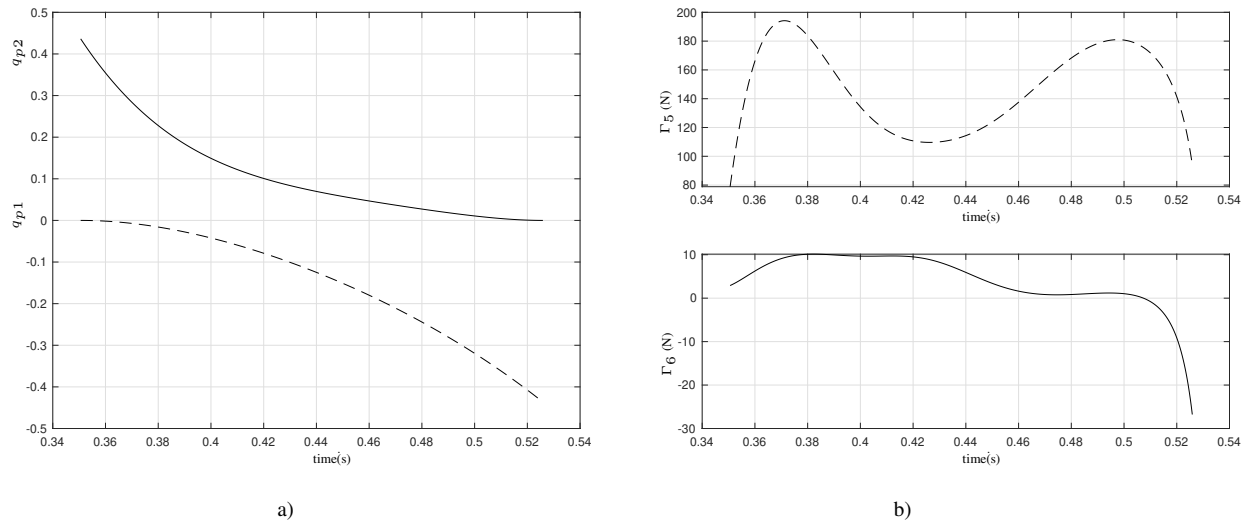


Figure 3: Front leg (solid lines) and rear leg (dashed lines) in double-support phase: a) Orientation angles of feet . b) Ankle torques.

**Conclusions**

The definition of a walking with a distributed in time double support is a complex problem, [4]. In the papers [1–3], we studied ballistic single-support motion coupled with *instantaneous* double-support phase. The torques in the biped joints at the instantaneous double support are impulsive one. The *distribution in time* of the double support allows to get a cyclic gait with the torques of finite magnitude. The future objective is to increase the time of the single-support motion and decrease the time of double-support motion in order to get biped walking closer to human one.

**References**

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